

Show that $1 + 2 + 3 + \dots + n = \frac{1}{2}n(n+1)$, for $n \geq 1$

i) basic induction

$$n=1$$

$$1 = \frac{1}{2}(1)(1+1) = \frac{1}{2} \cdot 1 \cdot 2$$

$$1 = 1$$

claim is proven for $n=1$

ii) step induction

$$1 + 2 + 3 + \dots + n = \frac{1}{2}n(n+1)$$

is claimed to be true then

$$1 + 2 + 3 + \dots + n + (n+1) = \frac{1}{2}(n+1)(n+2)$$

proving

$$[1 + 2 + 3 + \dots + n] + (n+1) = \left[\frac{1}{2}n(n+1) \right] + (n+1)$$

$$= \left(\frac{1}{2}n + 1 \right) (n+1)$$

$$= \frac{1}{2}(n+2)(n+1) \rightarrow \text{proven}$$

iii) Since basic & step inductions are correct

then $1 + 2 + 3 + \dots + n = \frac{1}{2}n(n+1)$